Циркуляры Главной Астрономической Обсерватории в Пулкове

№ 4 **Август 1932**

August 1932

POULKOVO OBSERVATORY CIRCULAR

№ 4

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On the Temperatures of the Nuclei of planetary Nebulae

In the present paper a new method for the determination of the temperatures of the nuclei of planetaries based on relative intensities of H and He^{+} lines is proposed.

Our method makes use of the fact that for each pair of definite quantum states (n, m) the ratio of Einstein's probability coefficients of spontaneous transition $A_{m}^{n'}$ for He^{+}, and A_{m}^{n} for H is constant and equal to 16 (See Appendix I)

$$\frac{A_m^{n'}}{A_m^{n}} = 16 \quad \ldots\ldots (1)$$

When estimating the temperatures of the nuclei of planetaries Zanstra had supposed, that all quanta emitted from the star having frequencies greater than ν_0 (the frequency of the limit of Lymann series) are absorbed by hydrogen atoms in nebulae. However, the nebulae contain so many atoms of He^{+} that we can assume that the radiation beyond the series limit of He^{+} ($4\nu_0$) is absorbed by He^{+} atoms (we neglect the line absorption). Therefore, only the radiation with frequencies between ν_0 and $4\nu_0$ is absorbed by hydrogen. Let the number of quanta with $\nu > 4\nu_0$ be N'_{ul}, and with $\nu_0 < \nu < 4\nu_0$ be N_{ul}. According to Planck's law

$$\frac{N'_{ul}}{N_{ul}} = \frac{\int_{4x_0}^{\infty} \frac{x^2}{e^x - 1}\,dx}{\int_{x_0}^{4x_0} \frac{x^2}{e^x - 1}\,dx}; \qquad x_0 = \frac{h\nu_0}{kT} \quad \ldots\ldots (2),$$

where T is the surface temperature of the star, h and k have their usual meaning.

At the same time N'_{ul} and N_{ul} are the numbers of ionizations and recombinations of He^{+} and H atoms per second. Let further

$$p_1 N_{ul},\ p_2 N_{ul},\ p_3 N_{ul}, \quad \ldots\ldots (3)$$

be the numbers of recombinations of H per second, by which the free electron jumps immediately into 1st, 2nd, 3rd qnuatum states.

Let

$$p'_1 N'_{ul},\ p'_2 N'_{ul},\ p'_3 N'_{ul}, \ldots \qquad (3')$$

be the corresponding numbers for He^+.

We have

$$\sum_i p_i = 1; \qquad \sum_i p'_i = 1 \ldots \qquad (4),$$

and neglecting the variation of p_i corresponding to the variation of temperature (See Appendix II).

$$p_i = p'_i; \qquad (i = 1, 2, 3, \ldots) \ldots \qquad (5).$$

The number $\mathfrak{N}_{nm}$ of each type of atomic transitions $n \to m$ per second is a linear homogeneous function of $p_i N_{ul}$ (J. A. Carroll, M. N., *90*, 588, 1930, our $\mathfrak{N}_{nm}$ have the same meaning as Carroll's $A_{nm}\ N_n$).

$$\mathfrak{N}_{nm} = \sum_i P^i_{nm} p_i N_{ul}; \qquad \mathfrak{N}'_{nm} = \sum_i P^{i'}_{nm} p_i N'_{ul} \ldots \qquad (6).$$

The coefficients P^i_{nm} are homogeneous functions of degree 0 of Einstein's coefficient A_{nm}. (Carroll's N_n are homogeneous functions of degree — 1). This function is the same for H, He^+ and for any atom with a homologous spectrum. From Euler's theorem and (1) we conclude

$$P^i_{nm} = P^{i'}_{nm} \ldots \qquad (7).$$

From the equations (6) and (7) we have

$$\frac{\mathfrak{N}'_{nm}}{\mathfrak{N}_{nm}} = \frac{N'_{ul}}{N_{ul}} \ldots \qquad (8).$$

Applying this equation to the transition $4 \to 2$, we have

$$\frac{\mathfrak{N}'_{42}}{\mathfrak{N}_{42}} = \frac{N'_{ul}}{N_{ul}} \ldots \qquad (8')$$

According to the definition of Einstein's probability coefficient we have

$$\mathfrak{N}'_{42} = N'_4 A'_{42}; \qquad \mathfrak{N}'_{43} = N'_4 A'_{43} \ldots \qquad (9),$$

where N'_4 is the number of He^+ atoms in 4th quantum state. Hence

$$\mathfrak{N}'_{42} = \frac{A_{42}}{A_{43}} \mathfrak{N}'_{43} \ldots \qquad (10).$$

Introducing (10) into (8′) we obtain

$$\frac{\mathfrak{N}'_{43}}{\mathfrak{N}_{42}} = \frac{A_{43}}{A_{42}} \frac{\mathfrak{N}'_{ul}}{\mathfrak{N}_{ul}} \ldots \qquad (11),$$

In the absence of absorption the ratio of intensities of 4686 and H_β is

$$\frac{J'_{43}}{J_{42}} = \frac{\mathfrak{N}'_{43}}{\mathfrak{N}_{42}} \frac{h\nu'_{43}}{h\nu_{42}} = \frac{\mathfrak{N}'_{43}}{\mathfrak{N}_{42}} \frac{\lambda_{42}}{\lambda'_{43}} \qquad (12),$$

where λ's are the wavelengths of the corresponding emission.

From (11) and (12) we find

$$\frac{N'_{ul}}{N_{ul}} = \frac{A_{42}}{A'_{43}} \frac{\lambda'_{43}}{\lambda_{42}} \frac{J'_{43}}{J_{42}} \qquad (13).$$

The ratio $\frac{J'_{43}}{J_{42}}$ may be obtained from measurements of intensities of 4686 and H_β. Therefore we also have the value $\frac{N'_{ul}}{N_{ul}}$ from observation. The following Table 1 contains the values of $\frac{N'_{ul}}{N_{ut}}$ for different values of T. The table is calculated according to (2). Taking $\frac{N'_{ul}}{N_{ul}}$ from observations we find T from this table.

Table 1

x_0	T	N'_{ul}/N_{ul}
0.0	∞	∞
0.2	785.000°	9
0.3	523.000	4.5
0.6	262.000	1
1.0	157.000	0.3
1.5	105.000	0.075
2.0	78.500	0.02
2.5	62.800	0.005

Applications to the planetary nebulae NGC 7009 and 7027. In the second column of table 2 the ratio $\frac{4686}{H_\beta}$ is given for two nebulae according to Berman's observations (L. O. B. 15, 86).

Table 2.

Nebulae	$\frac{4686}{H_\beta}$	T
7009	2ᵐ.18	115.000°
7027	1ᵐ.03	165.000°

The test of the method. The general form of the equation (13) is

$$\frac{N'_{nl}}{N_{nl}}=\frac{A_{nm}}{A'_{nk}}\,\frac{\lambda'_{nk}}{\lambda_{nm}}\,\frac{J'_{nk}}{J_{nm}}\qquad\qquad(14).$$

Applying this equation to the lines: 4686 and H_β, 4542 and H_η, 4200 and H_2 we obtain

$$\frac{A_{42}}{A'_{43}}\,\frac{\lambda'_{43}}{\lambda_{42}}\,\frac{J'_{43}}{J_{42}}=\frac{A_{92}}{A'_{94}}\,\frac{\lambda'_{94}}{\lambda_{92}}\,\frac{J'_{94}}{J_{92}}=\frac{A_{11,2}}{A'_{11,4}}\,\frac{\lambda'_{11,4}}{\lambda_{11,2}}\,\frac{J'_{11,4}}{J_{11,2}}$$

For the pairs under consideration the factors $\frac{A_{nm}}{A'_{nk}}\frac{J'_{nk}}{J_{nm}}$ are nearly the same. Therefore:

$$\frac{J'_{43}}{J_{42}}=\frac{J'_{94}}{J_{92}}=\frac{J'_{11,4}}{J_{11,2}}$$

According to Berman, we have correspondingly the following differences of brightness for the nebula NGC 7027:

$1^{m}.03$, $0^{m}.86$, $0^{m}.87$

The deviations from mean value $0^{m}.91$ are comparatively small, and thus the agreement of the theory with observations seems to be very satisfactory.

Appendix I. From the Schrödinger's equation for H and He^+ atoms it follows, that the normalized eigenfunctions φ'_n of He^+ are connected with the corresponding eigenfunctions φ_n of H according to the relation

$$\varphi_n'(x, y, z)=\sqrt{8}\,\varphi_n(2x, 2y, 2z)$$

The matrix elements q'_{nm} of He^+ atom are

$$q'_{nm}=\iiint_{-\infty}^{\infty}\varphi'_n\bar{\varphi}'_m\,x\,dx\,dy\,dz=\frac{1}{2}\iiint_{-\infty}^{\infty}\varphi_n(\xi,\eta,\zeta)\,\bar{\varphi}_m(\xi,\eta,\zeta)\,\xi\,d\xi\,d\eta\,d\zeta=\frac{1}{2}q_{nm}.$$

where q_{nm} are the corresponding matrix elements of H atom.

The Einstein probability coefficient for H atom has the form:

$$A_{nm}=C\nu^3|q_{nm}|^2$$

and for He^+ atom

$$A'_{nm}=C\nu'^3|q'_{nm}|^2$$

Since $\nu'=4\nu$ and $q'_{nm}=\frac{1}{2}q_{nm}$, we may write

$$A'_{nm}=16\,C\nu^2|q_{nm}|^3=16A_{nm}.$$

Appendix II. Applying (1) to the probabilities of the binding of a free electron with energy ε in a particular quantum state m, we have

$$A'_m(\varepsilon) = 16\, A_m\left(\frac{\varepsilon}{4}\right).$$

Therefore the ratio of the number of transitions of free electrons into the state m to the number of all recombinations is given by

$$p'_m(T) = \frac{p'_m N_{ul}}{N_{ul}} = \frac{\int A'_m(\varepsilon)\, e^{-\frac{\varepsilon}{kT}} \varepsilon^{1/2}\, d\varepsilon}{\sum\limits_m \int A'_m(\varepsilon)\, e^{-\frac{\varepsilon}{kT}} \varepsilon^{1/2}\, d\varepsilon} = \frac{\int A_m\left(\frac{\varepsilon}{4}\right) e^{-\frac{\varepsilon}{kT}} \varepsilon^{1/2}\, d\varepsilon}{\sum\limits_m \int A_m\left(\frac{\varepsilon}{4}\right) e^{-\frac{\varepsilon}{kT}} \varepsilon^{1/2}\, d\varepsilon}$$

if the velocity distribution of electrons is in accordance with Maxwell's law.

Introducing $\frac{\varepsilon}{4} = \varepsilon'$ we find

$$p'_m(T) = \frac{\int A_m(\varepsilon')\, e^{-\frac{4\varepsilon'}{kT}} \varepsilon'\, d\varepsilon'}{\sum\limits_m \int A_m(\varepsilon')\, e^{-\frac{4\varepsilon'}{kT}} \varepsilon'\, d\varepsilon'} = p_m\left(\frac{T}{4}\right).$$

Neglecting the variation of p_m with temperature, we may put

$$p'_m = p_m$$

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